

## Polymer Science 2025/26

### Exercise 11

1. The true stress,  $\sigma_r$ , as a function of the strain,  $\varepsilon$ , for a certain polymer is given by

$$\sigma_r = \sigma_0 \varepsilon^b (1 + \varepsilon^{-0.99b})$$

where  $\sigma_0 = 100$  MPa and  $b = 5$ . Determine the strain at which necking begins during a tensile test on this polymer? Is this necking process stable or unstable? Briefly explain your reasoning.

Tip 1: it is much easier to solve this problem graphically, rather than analytically. You can use tools such as Excel, Origin, etc.

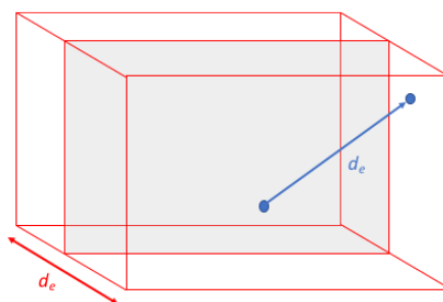
Tip 2: use the Considère criterion (Slide 320). For the required differentiation of  $\sigma_r$  with respect to  $\varepsilon$  (or equivalently  $\lambda$ ), use the product rule.

2. The yield strength of a polyethylene (PE) sample tested at 23 °C and at 0.001 s<sup>-1</sup> is 30 MPa in uniaxial tension and 31.5 MPa in uniaxial compression. Assuming that the yield strength,  $\sigma_y$ , is a linear function of the hydrostatic pressure  $p$ , calculate the yield strength in uniaxial tension when an external hydrostatic pressure of 500 MPa is applied.

Note: the external pressure is added to the material's existing stress state.

Tip: hydrostatic pressure is compressive in nature, so it is negative for tensile deformation and positive for compression.

3. Consider a cuboid volume of an isotropic entangled polymer with an area equal to 1 and a thickness of  $d_e$ . How many entanglement points are in this volume according to the model of the entanglement network?



- (i) Two entanglement points linked by a subchain are separated by a vector  $\vec{d}_e$  whose root mean square length is equal to  $d_e$ . How many of these sub-chains pass through the inner surface unit of the polymer?
- (ii) If  $U$  is the energy required to break a subchain and  $\gamma$  is the van der Waals surface energy of the polymer, what is the effective surface energy,  $\Gamma$ , when creating the voids of a craze at temperatures  $T \ll T_g$ ?
- (iii) Sketch the surface stretching mechanism for craze widening.
- (iv) The rate of movement of the polymer from the heads of the voids to the bases of the craze fibrils and therefore the craze expansion rate,  $v$ , are proportional to  $\nabla P^n$ , where  $n$  is an empirical constant and

$$\nabla P \approx \frac{P_2 - P_1}{D_0} = \frac{\sigma - \frac{4\Gamma}{D_0}}{D_0}$$

is the pressure gradient that drives the polymer from the head of the voids towards the base of the fibrils.  $\sigma$  is the applied stress, and  $D_0$  is the spacing of the fibrils. Show that the maximum craze widening speed for a given value of  $\sigma$  is obtained when

$$D_0 = \frac{8\Gamma}{\sigma}$$

and therefore, that the critical stress for widening of a craze at a speed  $v$  is

$$\sigma_c \propto \Gamma^{1/2} v^{1/2n}$$

- (v) Explain why polystyrene (PS) shows a much more fragile behavior than polycarbonate (PC), when deformed in tension.
- (vi) Is the craze formation via a disentanglement mechanism rather favored by (i) a low molar mass, (ii) a high strain rate, or (iii) high temperature? Explain!